Cash Generation in Business Operations: Some Simulation Models

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CASH GENERATION IN BUSINESS OPERATIONS: SOME SIMULATION MODELS

MORRIS BUDIN AND A. T. EAPEN*

I. INTRODUCTION

MANY EMPIRICAL STUDIES of the demand for money by business firms have been undertaken; yet the relevant constraints on the money balances and, in particular, the importance of trade credit policy (both accounts receivable and accounts payable), inventory policy, and the rate of change in the volume of production and sales in the determination of cash balances held by them have seldom, if ever, been adequately considered.

Both the Baumol and Tobin models[1] of the transactions demand for cash by business firms during a planning period allow for inflow of cash only at the start of the period; both models preclude any further inflow of cash until the beginning of the next cash planning period; both models assume disbursements at a uniform rate throughout a planning period. To be sure, both these models are very unrealistic; in actual practice, a business firm is very likely to have a series of inflows as well as outflows during a cash planning period. A firm's need for cash is obviously a function of the lack of correspondence between the amounts and times of receipts and disbursements—a perfect synchronization reduces average cash balances held by a firm to very minimal amounts.

This study proposes to investigate the net flow of cash generated in the course of business operations in a cash planning period and how such net inflows are affected by changes in policies concerning trade credit and inventories and also by changes in the rate of growth of production and sales. The net generation of cash during a planning period is, of course, the result of inflows and outflows caused by transactions in the past as well as the current periods. Undoubtedly the pattern of net cash flow generations during a planning period is a very important factor determining not only the magnitude and duration of required outside financing during that period but also the average cash balance that a firm holds during that period.

II. SIMULATION MODELS OF MONEY FLOWS

A number of money flow models, each simulating the cash planning process in a firm, will be considered in this section. The simulations are designed to follow the cash flows related to production and sales activities, omitting

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1. W. Baumol, "The Transactions Demand for Cash: An Inventory Theoretic Approach," *Quarterly Journal of Economics* 66 (November 1952), pp. 545-56; and J. Tobin, "The Interest Elasticity of Transactions Demand for Cash," *Review of Economics and Statistics* 38 (August 1956), pp. 241-47.

investment in equipment, financial flows due to bank loans, or security transactions. Whereas the models suggested by Baumol, Tobin, and others have been designed to explain transfers between cash and less liquid assets, the models presented here are concerned with the factors influencing cash generation arising from business operations in the short run.

The models in this paper have been restricted to the deterministic form; however, stochastic elements can be introduced. It appears doubtful that introducing purely randomized impacts into some deterministic models adds much to a clarification of our understanding of money flows. Most firms engage in such a large number of transactions in any one time period that the standard error around the expected value of inflows would be small. If a few large accounts dominate the accounts receivables, their actions would have such a major effect on the flows that a financial manager would give special attention to them. Furthermore, unexpected changes in money flows that are large would probably stimulate responses of a particular, not a random, nature. Small random variations of inflows might be compensated for by slight delays or accelerations of certain outflows; however, systematic changes in trade credit or inventory policies, and methods for handling cash flows, are probably far more significant in constructing a cash budget. Furthermore, the pattern of outflows of cash may be a function of the inflow pattern as well as other variables. Rather than having the two flows vary stochastically, the model would require a complex relationship between the two. For the models considered herein, it is assumed that no deviations occur during a planning period.

The Continuous Growth Model

The assumptions made for the simulation models are:

1. A cash budget is planned for a particular period or periods in the future. The financial officer considers production and sales patterns as fundamental predictors and uses these variables in formulating his budget. In the models cash flows will be related to dollar volumes of production and sales. Trade credit and inventory policies are also considered as major variables.
2. Money flows in any period occur from sales or purchases in past as well as current periods.
3. Wages for a period (or sub-period) must be paid at the end of the period (or sub-period).
4. Fixed costs and taxes must be paid at the end of the particular period (or sub-period) in which they are due.
5. The financial officer assumes a specified pattern of time lags between purchases and payments and between sales and receipts, which applies to the budget planning period. The pattern may vary among periods, but is assumed fixed for a particular period.
6. The model analyzes the net generation of cash during a particular planning period due to the inflows and outflows caused by transactions in the past as well as in the planning period. If the planning period is extremely short (e.g., a day), and if the net cash generation is zero, then the synchronization of inflows and outflows of cash for current activities can be assumed to be perfect. However, if longer planning periods are used (e.g., in these simula-

tions, one month), the intra-period synchronization may not be perfect and temporary shortages of cash might develop even though the net cash generation for the entire planning period is positive. The simulations below are based upon a planning period of one month to conform more closely to reality.

a. *The simplest money flow model: No carry-over of credit or inventories and a production-sales lag of n periods:*

$$C_n = Q_{o,n} - r\,Q_{t,n} - w\,Q_n,$$

$$C_{n+1} = Q_{1,n+1} - r\,Q_{t+1} - w\,Q_{n+1}$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$C_{n+m} = Q_{m,n+m} - rQ_{t+m,n+m} - wQ_{n+m} - T[\sum_o^m Q - r\sum_o^m Q - w\sum_o^m Q - F - D]$$

$$- \alpha(1 - T)\ [\sum_o^m Q - r\sum_o^m Q - w\sum_o^m Q - F - D] - F$$

If the system has no cash available at the outset, then C_n is the cash available at the end of the n^{th} sub-period due to flows in the n^{th} sub-period. Similarly, C_{n+1} is the cash available due to flows in the $(n + 1)^{th}$ period.

$Q_{o,n}$ = dollar value of sales paid to the firm in the n^{th} sub-period for production in sub-period zero (o^{th} sub-period).

$Q_{t,n}$ = dollar value of production of the t^{th} sub-period. The raw materials purchased for $Q_{t,n}$ are paid for in the n^{th} sub-period.

Q_n = dollar value of output of the n^{th} sub-period.

r = The ratio of the cost of raw materials required in production to the market value of the corresponding output (assumed fixed for the budget planning period).

w = wage ratio in terms of the market value of output (assumed fixed for the budget planning period).

T = tax rate on profits. (T = 0 if profit is zero or negative.)

F = fixed costs for the period that are paid at the end of the period.

D = depreciation permitted by tax laws.

α = per cent of net profits distributed as cash dividends at the end of the period. $\alpha = 0$ if profit is zero or negative in the simulations used in the paper, but need not be if firms prefer to distribute from previous earnings.

The subscripts are time indices where for simplicity we assume n follows o and t. A purchase in t is billed in n to maintain the simplicity of this first model.

If the planning period for cash is four sub-periods, then the cash available at the end of n + 3 is:

$$C_n + C_{n+1} + C_{n+2} + C_{n+3} = Q_{o,n} - r\,Q_{t,n} - w\,Q_n + Q_{1,n+1} - r\,Q_{t+1,n+1}$$
$$- w\,Q_{n+1} + Q_{2,n+2} - r\,Q_{t+2,n+2} - w\,Q_{n+2} + Q_{3,n+3}$$
$$- r\,Q_{t+3,n+3} - w\,Q_{n+3} - T\sum_{i=0}^{3} Q_{i,n+i}\,(1 - r - w - f - d) \qquad (1)$$

$$- \alpha(1 - T) \sum_{i=0}^{3} Q_{i,n+i} (1 - r - w - f - d) - f \sum_{i=0}^{3} Q_{n+i}$$

To simplify, the fixed costs and depreciation allowances are made proportional to output:

$$F = f \sum_{i=0}^{3} Q_{i,n+1} \quad \text{and} \quad D = d \sum_{i=0}^{3} Q_{i,n+1}.$$

Equation (1) reduces to

$$\begin{aligned} C_n + \ldots + C_{n+3} = \sum_{i=0}^{3} Q_{i,n+i} - r \sum_{i=0}^{3} Q_{t+i,n+i} - w \sum_{i=0}^{3} Q_{n+i} \\ - T \sum_{i=0}^{3} Q_{i,n+i} (1 - r - w - f - d) \\ - \alpha(1 - T) \sum_{i=0}^{3} Q_{i,n+i} (1 - r - w - f - d) \\ - f \sum_{i=0}^{3} Q_{n+i}. \end{aligned} \tag{2}$$

If the system is stationary, then

$$\sum_{i=0}^{3} Q_{i,n+1} = \sum_{i=0}^{3} Q_{t+i,n+i} = \sum_{i=0}^{3} Q_{n+i} = \Sigma Q,$$

and the system reduces to

$$\begin{aligned} C_n + \ldots + C_{n+3} = \Sigma Q - r\Sigma Q - w\Sigma Q - T\Sigma Q(1 - r - w - f - d) \\ - \alpha(1 - T)\Sigma Q(1 - r - w - f - d) - f\Sigma Q. \end{aligned}$$

If the firm is profitable and distributes all profits ($\alpha = 1$), then $C_n + \ldots + C_{n+3} = d\Sigma Q$ (i.e., the cash flows are equal to depreciation flows). If the firm suffers losses and $\alpha = 0$ then cash position will be negative if loss exceeds $d\Sigma Q$. Clearly for values of $\alpha < 1$ with profits, present cash exceeds $d\Sigma Q$.

If sales are declining, then

$$\sum_{i=0}^{3} Q_{n+i} < \sum_{i=0}^{3} Q_{i,n+i} \quad \text{and} \quad \sum_{i=0}^{3} Q_{n+i} < \sum_{i=0}^{3} Q_{t+i,n+i} \text{ (if t precedes n).}$$

Let us also assume $\alpha = 1$ if profits are present but zero if profits are zero or negative. Since raw materials constitute only a part of the value of the output, $r < 1$. Applying these to equation (2) and assuming profits as zero or positive, then the cash position of the firm will be positive as sales decline. If losses are incurred the positive or negative value of cash will depend on the rate of decline of sales and the values of r, w, and f.

During periods of growth no ready solution is evident, but the values to which the ratios of cash generated to sales converge under continuous growth will be solved in a later section where the complete model is used. However, it is evident that the ratio of cash generated to sales tends to decline as growth rates increase. This is due to the changes in the denominator of the C/Q fraction. Thus, variation in C/Q need not indicate changes in efficiency in the handling of cash or variation in trade credit. [See equation (3).]

$$\frac{C_n + \ldots + C_{n+3}}{\sum_{i=0}^{3} Q_{i,n+i}} = 1 - r\left(\frac{\sum_{i=0}^{3} Q_{t+i,n+i}}{\sum_{i=0}^{3} Q_{i,n+i}}\right) - w\left(\frac{\sum_{i=0}^{3} Q_{n+i}}{\sum_{i=0}^{3} Q_{i,n+i}}\right)$$

$$- [T + \alpha(1 - T)][1 - r - w - f - d] - f\left(\frac{\sum_{i=0}^{3} Q_{n+i}}{\sum_{i=0}^{3} Q_{i,n+i}}\right). \qquad (3)$$

Now if $\alpha = 1$ (profits are present) this reduces to

$$\frac{C_n + \ldots + C_{n+3}}{\sum_{i=0}^{3} Q_{i,n+i}} = 1 - r\left(\frac{\sum_{i=0}^{3} Q_{t+i,n+i}}{\sum_{i=0}^{3} Q_{i,n+i}}\right) - (f + w)\left(\frac{\sum_{i=0}^{3} Q_{n+i}}{\sum_{i=0}^{3} Q_{i,n+i}}\right)$$

$$- [1 - r - w - f - d] = r\left(1 - \frac{\sum_{i=0}^{3} Q_{t+i,n+i}}{\sum_{i=0}^{3} Q_{i,n+i}}\right)$$

$$+ (w + f)\left(1 - \frac{\sum_{i=0}^{3} Q_{n+i}}{\sum_{i=0}^{3} Q_{i,n+i}}\right) + d. \qquad (3a)$$

So the sign of the ratio of cash to sales depends on: (1) magnitudes of growth, if growth is positive; (2) the values of r and w; (3) the lag between production and sales; and (4) the lag between t and n. For periods of decline the growth ratios are less than unity, so the cash-sales ratio is positive, if profits are present.

If profits are equal to zero then $(1 - r - w - f - d) = 0$, and $T = 0$ for both cases. Also, if we assume both α and depreciation to be zero, then equation (3a) becomes:

$$\frac{C_n + \ldots + C_{n+3}}{\sum\limits_{i=0}^{3} Q_{i,n+i}} = 1 - r\left(\frac{\sum\limits_{i=0}^{3} Q_{t+i,n+i}}{\sum\limits_{i=0}^{3} Q_{i,n+i}}\right) - (f + w)\left(\frac{\sum\limits_{i=0}^{3} Q_{n+i}}{\sum\limits_{i=0}^{3} Q_{i,n+i}}\right). \tag{3b}$$

Under this condition positive growth means that the cash-sales ratio must be negative; if no growth occurs the ratio is zero; if growth is negative, the result is positive. If profits are negative, then the result depends on the magnitudes of growth, r, w, and f.

By using this simplified model it is possible to investigate the effects of reduced lags on cash flows. A direct comparison of the model in equations (2) or (3) using two different time lags brings out the implications of such a change.

Let us assume that a seven week lag is present between sales and cash inflows, and a five week lag between purchases of raw materials and cash outflows. To simplify, let us also assume no production-sales lag. The subscripts in the model below refer to dates as of the end of each sub-period. The cash budget is being planned for a four week period.

Now we have

$$C_8 = Q_1 - rQ_3 - wQ_8$$

. . . .

. . . .

$$\begin{aligned} C_{11} = {} & Q_4 - rQ_6 - wQ_{11} - F - T\,[\sum_8^{11} Q - r\sum_8^{11} Q - w\sum_8^{11} Q - F - D] \\ & - \alpha(1 - T)(\sum_8^{11} Q - r\sum_8^{11} Q - w\sum_8^{11} Q - F - D) \end{aligned}$$

The cash-sales ratio at the end of the four week period becomes

$$\begin{aligned} \frac{\sum_8^{11} C}{\sum_8^{11} Q} = {} & \frac{\sum_1^{4} Q}{\sum_8^{11} Q} - \frac{r\sum_3^{6} Q}{\sum_8^{11} Q} - \frac{w\sum_8^{11} Q}{\sum_8^{11} Q} - \frac{F}{\sum_8^{11} Q} \\ & - [T + \alpha(1 - T)](1 - r - w - f - d) \end{aligned} \tag{4}$$

Now let us assume that the seven week lag for inflows is reduced to five weeks; also, the five week lag for outflows is reduced to three:

$$C_8' = Q_3 - rQ_5 - wQ_8$$

. . . .

. . . .

$$C_{11}' = Q_6 - rQ_8 - wQ_{11} - F - T\,(\sum_8^{11} Q - r\sum_8^{11} Q - w\sum_8^{11} Q - F - D)$$

$$-\alpha(1-T)(\sum_{8}^{11} Q - r\sum_{8}^{11} Q - w\sum_{8}^{11} Q - F - D)$$

The cash-sales ratio is now:

$$\frac{\sum_{8}^{11} C'}{\sum_{8}^{11} Q} = \frac{\sum_{3}^{6} Q}{\sum_{8}^{11} Q} - \frac{r\sum_{5}^{8} Q}{\sum_{8}^{11} Q} - \frac{w\sum_{8}^{11} Q}{\sum_{8}^{11} Q} - \frac{F}{\sum_{8}^{11} Q} \tag{5}$$
$$- [T + \alpha(1-T)](1 - r - w - d - f)$$

It can be shown that the cash-sales ratios will be larger for (5) than (4) if the ratio of growth of sales is greater than zero. If the ratio is less than zero, the cash-sales ratio will be larger for (4) than (5).[2]

b. *The expanded deterministic model (includes accounts receivable, accounts payable, and inventories).*

The introduction of accounts receivable and accounts payable into the model adds stability to the evolving patterns by tying flows over past periods to actions taken in the current period. The general formulation of the cash flows is:

$$\begin{aligned}
C_n = {} & c_{o,n} Q_o + a_{-1,n}(1 - c_{-1,n-1}) Q_{-1} + a_{-2,n}(1 - a_{-2,n-2})(1 - c_{-2,n-2}) Q_{-2} \\
& + a_{-3,n}(1 - a_{-3,n-1})(1 - a_{-3,n-2})(1 - c_{-3,n-3}) Q_{-3} + \ldots \\
& + (a_{-i,n}) \ldots (1 - a_{-i,n-i+1})(1 - c_{-i,n-i}) Q_{-i} - r_o p_{o,n} Q_o \\
& - r_{-1} b_{-1,n}(1 - p_{-1,n-1}) Q_{-1} - r_{-2} b_{-2,n}(1 - b_{-2,n-1})(1 - p_{-2,n-2}) Q_{-2} \\
& - r_{-3} b_{-3,n}(1 - b_{-3,n-1})(1 - b_{-3,n-2})(1 - p_{-3,n-3}) Q_{-3} - \ldots - w_n Q_n
\end{aligned}$$

. . . .

. . . .

$$\begin{aligned}
C_{n+3} = {} & c_{3,n+3} Q_3 + a_{2,n+3}(1 - c_{2,n+2}) Q_2 + a_{1,n+3}(1 - a_{1,n+2})(1 - c_{1,n+1}) Q_1 \\
& + a_{o,n+3}(1 - a_{o,n+2})(1 - a_{o,n+1})(1 - c_{o,n}) Q_o + \ldots + \ldots - r_3 p_{3,n+3} Q_3 \\
& - r_2 b_{2,n+3}(1 - p_{2,n+2}) Q_2 - r_1 b_{1,n+3}(1 - b_{1,n+2})(1 - p_{1,n+1}) Q_1 - \ldots \\
& - \ldots - w_{n+3} Q_{n+3} - F - T\left[\sum_{i=0}^{3} Q_i (1 - r - w - f - d)\right] \\
& - \alpha(1-T)\left[\sum_{i=0}^{3} Q_i (1 - r - w - f - d)\right]
\end{aligned} \tag{6}$$

where: $c_{o,n}$, $c_{1,n+1}$ are per cents of outputs produced in the o^{th} and 1^{th} periods sold for cash in the n^{th} and $(n+1)^{th}$ periods respectively;

$a_{-1,n}$ is the per cent of payment received on sales made in the -1^{th} period and paid in the n^{th} period;

$(1 - c_{-1,n})$ is the accounts receivable at the outset of the n^{th} period from sales in the -1^{th} period;

r_o is the raw material ratio of the output of the o^{th} period;

$p_{o,n}$ is the per cent of purchases for the output of the o^{th} period paid for in the n^{th} period;

2. Proof is available upon request to the authors.

$(1 - b_{-1,n})\ (1 - p_{-1,n})$ is the unpaid portion (accounts payable) of the output of the -1^{th} period at the outset of the n^{th} period;

$b_{-1,n+1}$ is the per cent paid on the accounts payable of output of the -1^{th} period paid in the $(n + 1)^{th}$ period;

w = wage per unit of output.

The generalized formulation can be used to cumulate the sum of cash flows over a number of consecutive periods. If the period is as short as one day, the financial officer can identify the particular days he will lack cash. The parameters c, a, b, p, r, and w may all vary from one period to the next. In the tests below the concern is primarily with studying the convergence values of cash when a set of values are assigned to the parameters.

The model will be simplified to reduce the number of variables by assuming that the payment rate on receivables and payables is fixed for each period. That is, the firm pays X per cent of its open accounts payable of all prior periods, and collects Y per cent of open receivables of all prior periods.

Two variables representing inventory changes are now introduced to consider changes in inventory levels of finished goods and of raw materials. Z is the finished goods inventory variable and is somewhat artificially structured to fit the period-type-model. We assume that output of a particular period, n, is to be sold in period $n + t$. If the firm sells less than the amount produced, i.e., when inventory of finished goods is built up, $Z < 1$; if the firm sells more, i.e., inventory of finished goods is reduced, $Z > 1$; if $Z = 1$, no inventory change takes place. The same period-type assumption is made for raw materials purchases which are related to output of a particular period. But now when inventories of raw materials are being built up, $Z' > 1$; when inventories are being reduced, $Z' < 1$; no change, $Z' = 1$.

In the general formulation the inventory factor appears in equation (6) as:

$$\begin{aligned} C_n = {} & c_{o,n} Z_o Q_o + a_{-1,n} (1 - c_{-1,n-1}) Z_{-1} Q_{-1} + \ldots \\ & - r_o p_{o,n} Z'_o Q_o - r_{-1} b_{-1,n} (1 - p_{-1,n-1}) Z'_{-1} Q_{-1} - \ldots \end{aligned} \tag{7}$$

The model reduced to manageable proportions is presented below. A four period interval between output and sale is assumed; a similar lag between purchases and payments is maintained. A constant payment rate on all accounts is used. Finally, for simplification, all Z_i's are assumed equal; all Z'_i's are assumed equal; all r's are assumed equal. The flows are carried back for twenty-five weeks.

$$\begin{aligned} C_4 = {} & c Z Q_o + c(1 - c) Z Q_{-1} + c(1 - c)^2 Z Q_{-2} \\ & + \ldots + c(1 - c)^{20} Z Q_{-20} - p r Z' Q_o - p (1 - p) r Z' Q_{-1} \\ & - p(1 - p)^2 r Z' Q_{-2} - \ldots - p(1 - p)^{20} r Z' Q_{-20} - w Q_4 \end{aligned} \tag{8}$$

. . . .

. . . .

$$\begin{aligned} C_7 = {} & c Z Q_3 + c(1 - c) Z Q_2 + c(1 - c)^2 Z Q_1 + \ldots + c(1 - c)^{20} Z Q_{-17} \\ & - p r Z' Q_3 - p(1 - p) r Z' Q_2 - p(1 - p)^2 r Z' Q_3 - \ldots \\ & - p(1 - p)^{20} r Z' Q_{-17} - w Q_7 - \sum_4^7 F - T \, [\sum_0^3 Q - r \sum_0^3 Q - w \sum_0^3 Q \end{aligned}$$

$$-\sum_0^3 D - \sum_0^3 F] - \alpha(1-T)\,[\sum_0^3 Q - r\sum_0^3 Q - w\sum_0^3 Q - \sum_0^3 D - \sum_0^3 F].$$

The ratio of cash to sales for the four periods is:

$$\frac{\sum_4^7 C}{z\sum_0^3 Q} = \frac{cZ - pr\,Z'}{Z} + [c(1-c)\,Z - p(1-p)\,r\,Z']\,\frac{\sum_{-1}^{2} Q}{z\sum_0^3 Q} + \cdots$$

$$+ [c(1-c)^{20}\,Z - p\,(1-p)^{20}\,rZ']\,\frac{\sum_{-17}^{-20} Q}{z\sum_0^3 Q} - w\frac{\sum_4^7 Q}{z\sum_0^3 Q} - \frac{\sum_4^7 F}{z\sum_0^3 Q}$$

$$-\frac{T\sum_0^3 Q\,(1-r-w-f-d)}{z\sum_0^3 Q} - \frac{\alpha(1-T)\sum_0^3 Q\,(1-r-w-f-d)}{z\sum_0^3 Q} \qquad (8a)$$

$$(\text{where } \sum_0^3 D = d\sum_0^3 Q,\ \sum_0^3 F = f\sum_0^3 Q)$$

The equation for cash flows (8a) including accounts receivable, accounts payable and inventory changes has been simulated for various values of r, w, d, f, and for various trade credit and inventory policies. The rate of growth is an essential determinant, so the simulations have been run for alternative rates. In addition to the simulations based on constant rates of growth, simulations have been made of cyclical time series data.

III. Simulations to Establish Ranges for Cash Positions based on the Cash Flow Models

Several simulations have been run, for each of five rates of growth. These simulations seek to reflect the accounts receivable and accounts payable positions of firms over the 1947-1966 period. In the earlier years firms had a ratio of accounts receivable to quarterly sales of about 33 per cent; the accounts payable to quarterly sales ratio was slightly under 20 per cent. By 1966 these ratios had risen to nearly 50 per cent and 24 per cent, respectively. Where raw materials purchases constitute 50 to 60 per cent of the product, the accounts payable to quarterly raw materials purchases ratio changed from about 33 per cent in the late 1940's to about 45 per cent in the mid 1960's. The 33 per cent ratio of accounts payable to purchases (or accounts receivables to sales) is equivalent to about four weeks of purchases (or sales), the 45 to 50 per cent to about six weeks. To simulate these conditions under the assumptions that the "c" and "p" values in equation (8a) are constant for all periods, values of "c" and "p" were approximated to satisfy these ratios when all flow periods are being considered in weekly terms. For "c" and "p" equal to .20 and .14 we satisfy the 33 per cent, and 45 per cent to 50 per cent ratios, respectively, rather closely, so these have been used in most of the simulations. Examination of the data on individual firms revealed that the purchased materials ratios of 40 per cent to 60 per cent were most prevalent, so both have been used. To provide for the influences of inventory changes, Z and Z′ have been introduced for values of 0.9, 1.0, and 1.1. As the models are built on weekly flows, the increase or decrease of inventory of ten per cent of weekly production over the period of twenty weeks would result in a very high total accumulation. The purpose is to examine the monthly cash positions under these extremes, as well as more normal conditions. The growth rates range from 0.25 per cent to 10.0 per cent per week. Growth of over 0.5 per cent is rather extreme if maintained for twenty weeks consecutively.

The simulation uses a 50 per cent tax rate and an arbitrary cash dividend distribution rate of 50 per cent of net profits after taxes. The results summarized below indicate the convergence cash position for a firm that sold $1.00 of output twenty-five weeks ago and whose production rose by a particular percentage each week thereafter until the current week. The cash position is determined for various values of wages, fixed costs, and depreciation per dollar of output.

Some results of the simulations of the continuous growth model

The simulations provide nearly 70,000 results covering the conditions summarized below. The outputs are the convergence values of cash generated to sales ratios for the combination of conditions if they were to persist for a period of twenty-five weeks. Cyclical conditions and cash carry-overs from previous periods are not considered in the first group of simulations, as the purposes of the tests are to identify factors affecting cash generation and their magnitudes in a continuous growth system. Generally, the cash magnitudes generated in a continuous growth system with no cash carry-overs will be less than those generated in fluctuating systems with cash carry-overs, except in cases where the latter system experiences extremely rapid seasonal variations.

The simulations considered the following conditions in all combinations: (a) six growth rates per week (.0025, .0050, .0100, .0300, .0500, and .1000); (b) four credit conditions for accounts receivable (approximately six, five, four, and 3.5 weeks of sales outstanding in accounts receivable); (c) four credit conditions for accounts payable for the same periods as in (b); (d) three finished goods inventory accumulation conditions (increases—10 per cent of weekly production; decreases—10 per cent of weekly; no change); (e) three raw materials inventory accumulation, conditions as in (d); (f) three raw materials to market value of production ratios (.40, .50, and .60); (g) three wage to market value of production ratios (.10, .20, and .30); (h) three ratios of fixed costs to market value of production (.01, .05, and .10);

(i) three ratios of depreciation to market value of production (.01, .05, and .10); (j) the cash-sales ratios are in terms of four weeks of sales in the cash planning period. However, only a limited number of the results are included in this paper.

To exemplify the results, Table A provides the cash-sales ratios for one industry condition where the wage to production ratio is .30 and the raw material to production ratio is .50. The credit terms are varied for receivables and payables, and alternative inventory policies are compared. In addition, six alternative growth rates are also available for comparisons.

In comparing the results for alternative growth rates per week the relevant rates are generally .0025 and .0050, with exceptional cases at .0100 or more per week. The changes in the cash generated to sales ratios due to variations in growth rates are obviously influenced by trade credit and inventory policies also. However, in the change from .0025 to .0050 growth rate per week, the ratios change by only about 1.0 to 1.5 percentage points. For a change of growth from .0050 to .0100 the generation ratio rises by 1.5 to 2.0 percentage points.

At the extreme rates of growth, between .0500 and .1000, the ratio changes by 7.0 to 11.0 percentage points. For any given trade credit-inventory condition, the absolute decline in the cash-sales ratio is about one per cent per one-fourth of one per cent rise in growth rate.

Changes in trade credit policies have moderate impacts on cash-sales ratios. Comparing the extreme changes in trade credit policies for a particular inventory policy and growth condition indicates these results. For inventory policy $Z = 0.9$, $Z' = 1.0$, a shift in trade credit policy between the extremes ($AR = 6$ weeks, $AP = 3.5$ weeks to $AR = 3.5$ weeks, $AP = 6$ weeks) changes the ratio by over six percentage points for low growth rates, and by twelve percentage points for the highest growth rate. Alternative inventory policies cause some variations in the changes in the ratios, but these tend to be small. In general, shifts from a conservative payables policy (e.g., 3.5 weeks) combined with a liberal receivables policy (e.g., 6 weeks) to the reverse raises cash generated to sales ratios. The absolute increases are amplified by the growth rates. Credit policies vary slowly and cumulatively over time. Over the twenty years from 1947 to 1967, a shift of AR from four to six weeks of sales and AP from four to five weeks of sales has occurred in many industries. A shift of this magnitude increases cash generation ratios by 2.6 and 5.1 percentage points at the expected weekly growth rates (.0025 and .0050) and when inventory policies are $Z = 0.9$, $Z' = 1.0$ and $Z = 1.1$, $Z' = 0.9$ respectively.

Shifts in inventory policies have strong impact on cash-sales ratios. Rapid buildups and reductions in inventories may take place in fairly short periods of time. The changes in Table B reflect one such pattern. The shift from no inventory change ($Z = 1.0$, $Z' = 1.0$) to a 10 per cent of production weekly build up in finished goods and raw materials ($Z = .9$, $Z' = 1.1$) changes C/S by slightly over 17 percentage points at low growth rates and 10 percentage points at the highest growth rates. A build up of inventory of finished goods but no change in raw materials makes the C/S ratio move by 10 to 16

TABLE A
CASH GENERATED SALES (C/S) RATIOS FOR FIXED COSTS AT 1% AT VARIOUS GROWTH RATES
(R = .50, w = .30)

Accts Rec. (Sales in wks)	Accts Pay. (Sales in wks)	Inventory Policy for Finished Goods*	Inventory Policy for Raw Materials**	Depreciation Rates	GROWTH RATES PER WEEK					
					.0025	.0050	.0100	.0300	.0500	.1000
6	3.5	0.9	1.0	.05	−.066695	−.077756	−.098693	−.168920	−.222253	−.308967
6	3.5	0.9	1.1	.05	−.120393	−.130476	−.149530	−.213084	−.260890	−.337356
6	3.5	0.9	1.1	.01	−.153393	−.163476	−.182530	−.246084	−.293890	−.370356
6	3.5	1.0	0.9	.05	+.081800	+.068750	+.043986	−.039757	−.104235	−.211495
6	3.5	1.0	1.0	.05	+.032983	+.020822	−.002229	−.079906	−.139360	−.237302
6	3.5	1.0	1.1	.05	−.015833	−.027105	−.048444	−.120054	−.174484	−.263110
6	3.5	1.1	0.9	.05	+.157874	+.144234	+.119645	+.030511	−.037391	−.151125
6	3.5	1.1	1.0	.05	+.113938	+.101099	+.077589	−.005622	−.069004	−.174353
6	3.5	1.1	1.1	.05	+.070003	+.057964	+.035534	−.041756	−.100616	−.197579
6	5.2	0.9	1.0	.05	−.054100	−.063804	−.082271	−.145283	−.194530	−.278493
6	5.2	1.1	0.9	.05	+.167148	+.154507	+.131872	+.048449	−.016977	−.130115
4	4	0.9	1.0	.05	−.028950	−.037120	−.052820	−.108095	−.153436	−.236389
4	4	0.9	1.1	.05	−.082393	−.089541	−.103280	−.151645	−.191319	−.263902
4	4	1.0	1.0	.05	+.070852	+.061568	+.043726	−.019085	−.070608	−.164873
4	4	1.1	0.9	.05	+.197112	+.184085	+.165023	+.089720	+.029437	−.080852
3.5	6	0.9	1.0	.05	−.004808	−.010835	−.022644	−.066672	−.105816	−.185185
3.5	6	0.9	1.1	.05	−.056297	−.061165	−.070764	−.107184	−.140294	−.209153
3.5	6	0.9	1.1	.01	−.089297	−.094165	−.103764	−.140184	−.173294	−.242153
3.5	6	1.0	1.0	.05	+.093261	+.086004	+.071845	+.019686	−.025946	−.116735
3.5	6	1.1	0.9	.05	+.213933	+.204854	+.189300	+.123132	+.068083	−.038998
3.5	6	1.1	1.1	.05	+.129679	+.122494	+.109681	+.069470	+.011663	−.078218

* If the value is less than unity the inventory of finished goods is *increasing*.
** If the value is less than unity the inventory of raw materials is *decreasing*.

percentage points at alternative growth rates (see Table B). Reductions in raw material inventories have less of an impact than reductions in finished goods inventories. It is clear that changes in trade credit policy interact with inventory policy and growth rates to affect C/S.

IV. The Cyclical Model

Cyclical models, rather than continuous growth models, probably are more analogous to actual business conditions. As the present period's cash flows are in part dependent on past purchases, sales, and payments patterns, the cyclical movements produce a wide variety of results. A model used in this simulation has various seasonal fluctuations as well as an upward drift growth pattern built into it.[3] The seasonal fluctuations are fairly extreme with 5 and 10 per cent variations in weekly production in some months, to test the impact of such changes.

In the model, purchase of raw materials precede production by four weeks. To test extreme values, the model calls for accounts receivable at any time to total the past seven weeks of sales while accounts payable are the total of the past five weeks of purchases. Thus, the firm is more liberal in giving credit than in receiving it. Raw materials constitute 60 per cent of the sale value of the product, and to further emphasize the extreme condition, wages (not lagged) are 30 per cent of the product. Thus, gross profits are kept low at 10 per cent. Furthermore, fixed costs are 5 per cent, depreciation allowed under tax laws is 3 per cent, tax is 50 per cent of net profits, and the firm distributes half of the earnings after taxes. Clearly, the model is a near-zero profit model and would reflect rather extreme cash needs. The model assumes a gradual buildup of finished goods inventory prior to an upswing in sales and the working off of this inventory prior to the downswing in sales.

Under these conditions, the deterministic model using data on production and sales indicates that the cash generated to monthly sales ratio varies from —36.9 per cent to +56.9 per cent. The negative sign indicates cash shortages from operations; the positive sign indicates surpluses. The cash shortage period builds slowly at first as inventories of raw materials and finished goods are both increasing; as the inventory buildup continues, the cash needs reach a peak. The peak persists even when the inventory buildup slows down; the cash shortage is present but below the peak during the first two months when inventories are being reduced or kept stable.

The cash surplus ratio is moderate to low during the months preceding the production build up. These months do not immediately follow the past production surge. The cash surplus ratio crescendo occurs in the months immediately following the sales and production buildups as payments flow in.

The model may be redesigned to provide for less extreme, and probably more realistic, conditions by reducing the wage share to 20 per cent. If so, the range of the cash to monthly sales ratio is maximum negative —34.4 per cent. The reduction in wage costs is offset to a large extent by taxes and distribution of earnings due to the presence of larger profits.

3. This model is available upon request to the authors.

TABLE B

COMPARISON OF CASH GENERATED SALES (C/S) RATIOS FOR ALTERNATIVE INVENTORY AND CREDIT POLICIES AND GROWTH CONDITIONS
(Industry has R = .60, W = .30, F = .10 so profits are zero)

Accounts Receivable (Sales in weeks)	Accounts Payable (Sales in weeks)	Inventory: $Z = 1.0, Z' = 1.0$ Growth Rates per Week			Inventory: $Z = .9, Z' = 1.1$ Growth Rates per Week			Inventory: $Z = .9, Z' = 1.0$ Growth Rates per Week		
		.0025	.0100	.1000	.0025	.0100	.1000	.0025	.0100	.1000
6	6	−.0255	−.0500	−.2256	−.1946	−.2109	−.3279	−.1322	−.1526	−.2988
6	5.25	−.0359	−.0617	−.2406	−.2072	−.2252	−.3462	−.1437	−.1656	−.3155
6	4	−.0468	−.0755	−.2643	−.2206	−.2420	−.3751	−.1559	−.1809	−.3418
6	3.5	−.0496	−.0796	−.2749	−.2240	−.2470	−.3867	−.1590	−.1855	−.3524
5.25	6	−.0082	−.0305	−.2007	−.1773	−.1914	−.3029	−.1149	−.1331	−.2739
5.25	5.25	−.0186	−.0422	−.2157	−.1900	−.2057	−.3212	−.1265	−.1461	−.2905
5.25	4	−.0295	−.0560	−.2394	−.2033	−.2225	−.3502	−.1386	−.1614	−.3168
5.25	3.5	−.0323	−.0601	−.2489	−.2067	−.2275	−.3618	−.1417	−.1660	−.3274
4	6	+.0100	−.0075	−.1612	−.1590	−.1684	−.2634	−.0967	−.1101	−.2344
4	5.25	−.0003	−.0192	−.1761	−.1717	−.1827	−.2817	−.1082	−.1231	−.2511
4	4	−.0113	−.0330	−.1998	−.1851	−.1995	−.3107	−.1204	−.1384	−.2774
4	3.5	−.0140	−.0371	−.2094	−.1885	−.2046	−.3223	−.1235	−.1430	−.2880
3.5	6	+.0146	−.0006	−.1453	−.1544	−.1615	−.2476	−.0921	−.1033	−.2185
3.5	5.25	+.0042	−.0123	−.1602	−.1671	−.1758	−.2659	−.0136	−.1163	−.2352
3.5	4	−.0066	−.0261	−.1840	−.1805	−.1927	−.2948	−.1157	−.1316	−.2615
3.5	3.5	−.0094	−.0302	−.1935	−.1839	−.1977	−.3064	−.1188	−.1362	−.2721

V. THE MAX-NEGATIVE MODEL

The third model has been constructed to reflect the views that might be taken by the firm's finance officer who is planning to meet his cash flows. The model postulates an extremely conservative officer who makes the following assumptions: 1. The money flow stream is not continuous but is rather highly concentrated at particular pay-off dates. All sales are on a credit basis and he expects that the sales of the previous month will be paid in the current month. Under the worst condition none of these accounts receivable will be paid prior to the tenth of the month. 2. Similarly, all purchases were made on credit and the past month's accounts payable will be paid on the ninth day of the current month. 3. Two weeks of wages will have to be met before the tenth of the month. 4. He refuses to issue checks unless he has the cover in the bank..5 Sales = Output. (No inventory changes).

The maximum negative cash positions for (1) no growth between two months; (2) a 20 per cent growth; and (3) a 20 per cent decline are summarized in the accompanying Table C. Panels A indicate the wages-sales and raw materials-sales combination. Row B for each rate of growth indicates the required cash to monthly sales ratios to avoid cash shortages.

The model can be easily modified to include inventory changes.

This model is of interest since it allows study of the conditions under which cash shortages and surpluses occur, given an extremely conservative outlook. A policy of $C/S = 45$ per cent would imply shortages when for example when $w \cup r$ is $.10 \cup .50$, $.10 \cup .60$, etc.

For $C/S = 65$ per cent the combinations are reduced to $w \cup r$:

$.10 \cup .70$, $.10 \cup .80$, $.20 \cup .60$, $.20 \cup .70$, $.30 \cup .60$, $.40 \cup .50$.

For $C/S = 85$ per cent no cash would be required except for the 20 per cent growth rate where $w = .10$, $r = .80$.

Of these combinations those that have $w + r = 90$ per cent are probably rare in industry. Thus, a firm following a policy of a cash to annual sales ratio of 5 per cent (60 per cent on a monthly sales basis) runs very little risk in its cash needs even under these extreme conditions. Apparently, firms do operate as conservatively as the model implies, since cash to monthly sales ratios for many firms have run in the 60 per cent and higher range from 1947 to 1959, and only in recent years have they declined to the 40 per cent level and below.

VI. CONCLUSIONS

The models developed in this paper account for money flows arising from current operating activities, but do not consider balances held for investment (physical and financial) or for compensatory balance purposes. The significance of the models lies in clarifying the roles of growth, credit policy, inventory policies, and the raw material and wage structure of production in cash generation. Furthermore, the models stress that the time shape of past events which induce money flows in the cash budget plan period are extremely important. The major influences of the short run shifts in inventory policies, and the longer run gradual shifts in trade credit policies, can be identified in all of the models.

TABLE C

MAX-NEGATIVE MODEL SIMULATED RESULTS OF REQUIRED C/S

(A) Wage/Sales ∪ Raw Materials/Sales	.10∪.40 .30∪.30 .50∪.20 .70∪.10	.10∪.50 .30∪.40 .50∪.30 .70∪.20	.10∪.60 .30∪.50 .50∪.40	.10∪.70 .70∪.60	.10∪.80	.20∪.30 .40∪.20 .60∪.10	.20∪.40 .40∪.30 .60∪.20 .80∪.10	.20∪.50 .40∪.40 .60∪.30	.20∪.60 .40∪.50	.20∪.70
(B) No Growth: Required C/S	.45	.55	.65	.75	.85	.40	.50	.60	.70	.80

(A) Wage/Sales ∪ Raw Materials/Sales	.10 ∪ .40	.10 ∪ .60	.20 ∪ .30	.20 ∪ .50	.20 ∪ .70	.30 ∪ .30	.30 ∪ .50	.30 ∪ .60	.40 ∪ .10	.40 ∪ .30	.40 ∪ .50	.50 ∪ .10	.50 ∪ .20	.50 ∪ .40	.60 ∪ .10	.60 ∪ .30	.70 ∪ .10	.70 ∪ .20	.80 ∪ .10
(B) 20% Growth: Required C/S	.46	.66	.42	.62	.82	.48	.68	.78	.34	.54	.74	.40	.50	.70	.46	.66	.52	.62	.58
(B) 20% Decline: Required C/S	.44	.64	.38	.58	.78	.42	.62	.72	.26	.46	.66	.30	.40	.60	.34	.54	.38	.48	.42

The cyclical model indicates how the cash flows due to sales and purchases of previous periods affect the relationship between the cash-sales ratio and inventory changes. A shift from inventory buildup to stability and decline is not accompanied immediately by a rise in the cash-sales ratio. The ratio lags until the inventory reduction is in full swing. During the inventory build-up season the cash-sales ratio need not decline in the initial periods, but does in the subsequent ones. Thus a statistical relationship between these variables must consider these lagged effects.

From the continuous growth and cyclical models it is evident that for only brief periods, when inventory building patterns are strong and cash inflows from previous periods are restrained, should the firm hold its cash ratios at 30 to 40 per cent of monthly sales. During other periods, far smaller holdings would meet its needs. This analysis does not consider stochastic variations in the flows. However, it appears doubtful that these variations should require the firm to deviate from the suggested range by many percentage points. Furthermore, errors in estimating these flows can be compensated for by liquidating securities or by temporary borrowings. Policies which suggest holding some fixed per cent of expected current annual sales in cash are therefore misleading. A behavior pattern prevalent in the late 1960 period, wherein firms hold 2 per cent of annual sales in the cash form really signifies a policy of holding 24 per cent of average monthly sales liquid. This may be required for some months, but is certainly excessive for many months for most firms. Policies which suggest 5 per cent to 10 per cent of monthly sales as the cash position might well suffice.